2015

MATRICULATION EXAMINATION

DEPARTMENT OF MYANMAR EXAMINATION

MATHEMATICS **Time Allowed: (3) Hours**

WRITE YOUR ANSWERS IN THE ANSWER BOOKLET.

SECTION (A)

(Answer **ALL** questions. Choose the correct or the most appropriate answer for each question. Write the letter of the correct or the most appropriate answer.)

1. (1) Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x^2 - 1$ and $g(x) = 2^x$. If $(g \circ f)(k) = 8$, then $k =$

   A. 2 B. -2 C. ± 2 D. 5 E. ± 8

   (2) If a binary operation $\odot$ is defined by $p \odot q$ = the remainder when $2p + q$ is divided by 10, then $6 \odot 4 =$

   A. 3 B. 4 C. 5 D. 6 E. 7

   (3) Given n is an integer, the remainder when $x^{2n} - 4x + 7$ is divided by $(x + 1)$ is

   A. 14 B. 12 C. 10 D. -14 E. -12

   (4) If $x - 2$ is a factor of $x^n + 9x^2 - 68$, then $n =$

   A. 3 B. 4 C. 1 D. 2 E. 5

   (5) In the expansion of $\left(\dfrac{1}{2x^2} - x\right)^9$, the term independence of x is

   A. 4^{th} term B. 5^{th} term C. 6^{th} term D. 7^{th} term E. 8^{th} term

   (6) ${}^nC_r + {}^nC_{n-r} =$

   A. $2\,{}^nC_r$ B. ${}^{2n}C_r$ C. ${}^nC_{2n-1}$ D. ${}^nC_{n-2r}$ E. none of these

   (7) The solution set in R for the inequation $x^2 + 12 < 3$ is

   A. $\{x \mid x < \pm 3\}$ B. $\{x \mid x > 3 \text{ or } x < -3\}$ C. $\{x \mid -3 < x < 3\}$ D. $\varnothing$ E. R

   (8) In a certain sequence if $u_1 = 3$, $u_{n+1} = \dfrac{u_n}{n}$, then $u_4 =$

   A. 3 B. 2 C. $\dfrac{9}{2}$ D. $\dfrac{1}{2}$ E. $\dfrac{1}{4}$

   (9) Given that 3, x, y, z, 23, . . . is an arithmetic sequence, then $z =$

   A. 13 B. 18 C. 21 D. 22 E. none of these

   (10) If the arithmetic mean between 1 and x is 4 and the geometric mean between 2 and y is 6, then $x + y =$

   A. 7 B. 18 C. 25 D. 11 E. none of these

   (11) If $P = \begin{pmatrix} 1+2x \\ 10 \end{pmatrix}$, $Q = \begin{pmatrix} 2 \\ 1-y \end{pmatrix}$ and $P + 2Q = \begin{pmatrix} 3 \\ 2y \end{pmatrix}$, then $\dfrac{y}{x} =$

   A. 3 B. 2 C. -3 D. -2 E. -4

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(12) If $M = \begin{pmatrix} 2^x & -3 \\ -9 & 7 \end{pmatrix}$ and $\det M = 1$, then $M^{-1} =$

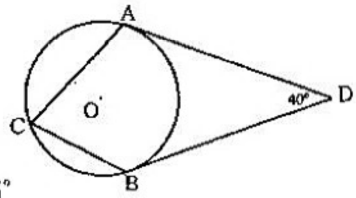
A. $\begin{pmatrix} 4 & -3 \\ -9 & 7 \end{pmatrix}$ B. $\begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix}$ C. $\begin{pmatrix} 7 & -3 \\ -9 & 4 \end{pmatrix}$ D. $\begin{pmatrix} 7 & 3 \\ 9 & 4 \end{pmatrix}$ E. $\begin{pmatrix} -7 & 3 \\ 9 & -4 \end{pmatrix}$

(13) A bag contains 12 balls of 2 red, 4 blue, and 6 white. If a draw is made, then the probability of getting blue or white is

A. $\frac{1}{4}$ B. $\frac{1}{6}$ C. $\frac{5}{6}$ D. $\frac{1}{2}$ E. 0

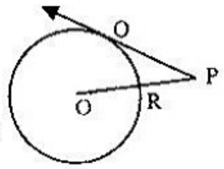
(14) A coin is tossed 2 times. the probability of getting at least one tail is

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{3}{4}$ D. 1 E. 0



(15) In figure, AD and BD are tangents to the circle whose centre is O. If $\angle ADB = 40^\circ$, then $\angle ACB$ is

A. 140° B. 70° C. 35° D. 55° E. 65°



(16) In circle O, PQ is a tangent at Q. If PQ = 4cm, PR = 2cm, then the length of the diameter is

A. 6cm B. 10cm C. 12cm D. 16cm E. 24cm

(17) In ΔABC, P and Q are two points on the sides AB and AC respectively. If $PQ // BC$ and $\alpha(\Delta APQ):\alpha(BCQP) = 9:16$, then AP : PB is

A. 3:4 B. 4:3 C. 3:5 D. 5:3 E. 3:2

(18) The position vector of A, B, C are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. If $\overline{AC} = -2\overline{CB}$, then $\vec{c}$ is

A. $-\vec{a}+2\vec{b}$ B. $\vec{a}-2\vec{b}$ C. $2\vec{a}+\vec{b}$ D. $2\vec{a}-\vec{b}$ E. $-2\vec{a}+\vec{b}$

(19) The map of the point (2, 0) which rotates through an angle of 90° about O in clockwise direction is

A. (2, – 2) B. (0, – 2) C. (0, 2) D. (– 2, 0) E. (– 2, 2)

(20) $\sin(270^\circ) + \cos(720^\circ)$ is

A. 0 B. – 1 C. 1 D. $\sqrt{2}$ E. none of these

(21) If θ is an acute angle and $\sin\theta = k$, then $\sin 2\theta$ is

A. $2k\sqrt{1-k^2}$ B. $k\sqrt{1-k^2}$ C. $2k\sqrt{k^2-1}$ D. $k\sqrt{k^2-1}$ E. $\sqrt{k^2-1}$

(22) In triangle ABC if $\alpha = 30^\circ$, $\gamma = 105^\circ$ and $b = 8$, then a =

A. $8\sqrt{3}$ B. $8\sqrt{2}$ C. $6\sqrt{2}$ D. $4\sqrt{3}$ E. $4\sqrt{2}$

(23) The gradient of the tangent to the curve $y = ax^2 - 4x + 3$ at the point x = 1 is – 2. The value of a is

A. 3 B. 2 C. 1 D. –3 E. 4

(24) The stationary point of the curve $y = x^2 - 4x$ is

A. (2, 4) B. (2, 0) C. (– 2, 12) D. (0, 4) E. (2, – 4)

(25) Given that $y = \frac{\ln x^2}{3x}$, the value of $\frac{dy}{dx}$ when x = 1 is

A. 1 B. $\frac{2}{3}$ C. $\frac{1}{3}$ D. – 1 E. – 2

(25 marks)

SECTION (B)

(Answer **ALL** questions)

2. If $f : R \to R$ is defined by $f(x) = x^2 + 3$, find the function g such that $(g \circ f)(x) = 2x^2 + 3$. (3 marks)

(OR)

Given that the expression $2x^3 + ax^2 + bx + c$ leaves the same remainder when divided by $x - 2$ or by $x + 1$, prove that $a + b = -6$. (3 marks)

3. The four angles of a quadrilateral are in A.P. Given that the value of the largest angle is three times the value of the smallest angle, find the values of all four angles. (3 marks)

(OR)

The second term of a G.P., is 64 and the fifth term is 27. Find the first 6 terms of the G.P. (3 marks)

4. In $\triangle ABC$, $\overline{BP} = \overline{PC}$ and $\overline{CQ} = \frac{1}{3}\overline{CA}$. Prove that $2\overline{BC} + \overline{CA} + \overline{BA} = 6\overline{PQ}$. (3 marks)

5. If $\alpha + \beta + \gamma = 180^\circ$, prove that $\sin\frac{\alpha+\beta}{2} = \sin\left(90^\circ + \frac{\gamma}{2}\right)$. (3 marks)

6. Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x^2 + 3x - 10}$ and $\lim_{x \to \infty} \frac{\sqrt{2x} - \sqrt{a}}{\sqrt{2x} + \sqrt{a}}$. (3 marks)

SECTION (C)

(Answer any **SIX** questions)

7.(a) The functions f and g are defined for real x by $f(x) = 2x - 1$ and $g(x) = \frac{2x+3}{x-1}$, $x \neq 1$.

Evaluate $(g^{-1} \circ f^{-1})(2)$. (5 marks)

(b) Let J^+ be the set of all positive integers. Is the function $\odot$ defined by $x \odot y = x + 3y$ a binary operation on J^+? If it is a binary operation, solve the equation $(k \odot 5) - (3 \odot k) = 2k + 13$. (5 marks)

8.(a) The expression $x^3 + ax^2 + bx + 3$ is exactly divisible by $x + 3$ but it leaves a remainder of 91 when divided by $x - 4$. What is the remainder when it is divided by $x + 2$? (5 marks)

(b) If the 2^{nd} and the 3^{rd} term in $(a + b)^n$ are in the same ratio as the 3^{rd} and 4^{th} in $(a + b)^{n+3}$, then find n. (5 marks)

9.(a) Use a graphical method to find the solution set of $x^2 \leq \frac{4}{5}(x+3)$, and illustrate it on the number line. (5 marks)

(b) The fourth term of an A.P. is 1 and the sum of the first 8 terms is 24. Find the sum of the first three terms of the progression. (5 marks)

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10.(a) The sum of the first three terms of a G.P. is 27 and the sum of the fourth, fifth and sixth terms is -1. Find the common ratio and the sum to infinity of the G.P.

(5 marks)

(b) Given that the matrix $A = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ and that $A^2 - kA + 5I = O$, find the value of k.

(5 marks)

11.(a) Find the solution set of the systems of equations

$\left.\begin{array}{l} 5x+6y=25 \\ 3x+4y=17 \end{array}\right\}$ by matrix method; the variables are on the set of real numbers.

(5 marks)

(b) The probabilities of students A, B, C to pass an examination are $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$ respectively. Find the probability that at least one of them will pass the examination.

(5 marks)

12.(a) Two circles cut at A, B. The tangent to the first at A meets the second again at C; and the tangent to the second at B meets the first again at D. Prove that AD and CB are parallel. (5 marks)

(b) ABC is a triangle in which AB = AC. P is a point inside the triangle such that $\angle PAB = \angle PBC$. Q is the point on BP such that AQ = AP. Prove that ABCQ is cyclic.

(5 marks)

13.(a) In ΔABC, AD and BE are the altitudes. If $\alpha(\Delta DEC) = \frac{3}{4}\alpha(\Delta ABC)$, prove that $\angle ACB = 30^\circ$. (5 marks)

(b) Find the matrix which will rotate 30° and then reflect in the line OY. What is the map of the point (1, 0)? (5 marks)

14.(a) Without the use of table evaluate $\tan(\alpha+\beta+\gamma)$, given that $\tan\alpha = \frac{1}{2}$, $\tan\beta = \frac{1}{3}$ and $\tan\gamma = \frac{1}{4}$. (5 marks)

(b) A, B, C are three towns, B is 10 miles from A in a direction N 47° E. C is 17 miles away from B in a direction N 70° W. Calculate the distance and direction of A from C.

(5 marks)

15.(a) If $y = \ln(\cos 2x)$, prove that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 4 = 0$. (5 marks)

(b) Determine the turning point on the curve $y = 2x^3 + 3x^2 - 12x + 7$ and state whether it is a maximum or a minimum. Then sketch the graph of the curve. (5 marks)