2018

MATRICULATION EXAMINATION

DEPARTMENT OF MYANMAR EXAMINATION

MATHEMATICS **Time Allowed: (3) Hours**

WRITE YOUR ANSWERS IN THE ANSWER BOOKLET.

SECTION (A)

(Answer **ALL** questions. Choose the correct or the most appropriate answer for each question. Write the letter of the correct or the most appropriate answer.)

1. (1) Let f and g be functions given by $f(x) = 2x$ and $g(x) = x + 3$. If $(g \circ f)^{-1}(t) = 1$ then $t =$

A. 3 B. -3 C. 5 D. -5 E. 2

(2) An operation $\odot$ is defined by $x \odot y$ is the remainder when x is divided by y, then

$(61 \odot 7) - (5 \odot 5) =$

A. -4 B. 4 C. 3 D. 5 E. 6

(3) It is given that the remainder is 178 when $x^n - 5x^2 - 20$ is divided by $x - 3$, then the value of n is

A. -4 B. 4 C. 3 D. -3 E. 5

(4) If $x - 2$ is a factor of $x^{2n+1} + 3x^2 - 44$, then the value of n is

A. 2 B. -2 C. 4 D. 1 E. -1

(5) The term independent of x in the expansion of $(\tan x + \cot x)^6$ is

A. 6 B. 15 C. 12 D. 18 E. 20

(6) The coefficient of x^n in the expansion of $(1+x)^{2n}$ is

A. ${}^{2n}C_{n-1}$ B. ${}^{2n}C_n$ C. ${}^{n}C_n$ D. ${}^{2n}C_{n+1}$ E. none of these

(7) The solution set of the inequation $\dfrac{1}{3x^2+2} \geq 0$ is

A. $\{x \mid x \leq 0\}$ B. $\varnothing$ C. $\{x \mid x < 0\}$ D. $\{x \mid x > 0\}$ E. R

(8) In a certain series, if $S_n = (n-1)(n-2)$ then $u_n =$

A. $4(n-1)$ B. $4(n-2)$ C. $2(2-n)$ D. $2(n-2)$ E. $2(n-1)$

(9) An an A.P., the fourth term is 6. Then $S_7 =$

A. 35 B. 40 C. 42 D. 45 E. 50

(10) If the A.M. between 3 and x is 6, then the positive G.M. between x and x + 7 is

A. 9 B. 12 C. 15 D. 18 E. 21

(11) If A is 2×2 matrix such that $\det A = k$ and p is real number then $\det(pA) =$

A. pk B. pk^2 C. p^2k D. p^2k^2 E. p

(12) The matrix $M = \begin{pmatrix} a & 4 \\ 16 & b \end{pmatrix}$ is singular and a, b are positive integers. Then a + b cannot be

A. 16 B. 20 C. 34 D. 48 E. 65

[P. T. O.

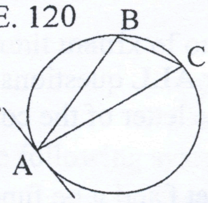
(13) A box contains 5 cards numbered as 2, 3, 4, 5 and 9. If a card is chosen, then the probability of getting not a prime number is

A. $\frac{2}{3}$ B. $\frac{3}{5}$ C. $\frac{2}{5}$ D. $\frac{1}{5}$ E. $\frac{1}{3}$

(14) A die is rolled 120 times. The expected frequency of a prime number is

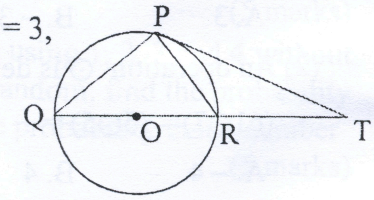
A. 40 B. 60 C. 80 D. 20 E. 120

(15) In the figure, AT is a tangent at A and $\angle ABC = 125°$, then $\angle TAC =$



A. 140° B. 125° C. 70° D. 80° E. 55°

(16) In the figure, PT is a tangent at P. PT = 6, PQ = 4 and PR = 3, then RT =



A. 3 B. 4 C. 5 D. 6 E. 7

(17) In ΔABC, D is a point on AB such that AD = 3DB. E is a point on AC such that DE // BC. If $\alpha(\Delta ADE) = 36$, then $\alpha(\Delta ABC) =$

A. 56 B. 64 C. 72 D. 80 E. 96

(18) If the position vectors of P and Q with respect to origin O are $-2\hat{i} + 7\hat{j}$ and $4\hat{i} - 5\hat{j}$ respectively and PR : RQ = 2 : 1, then $\overrightarrow{RQ} =$

A. $2\hat{i} - 4\hat{j}$ B. $-2\hat{i} + 4\hat{j}$ C. $3\hat{i} - \hat{j}$ D. $-3\hat{i} + \hat{j}$ E. $\hat{i} - 2\hat{j}$

(19) The map of the point (4, 0) by a rotation through an angle 180° about O in clockwise direction is

A. (4, – 4) B. (0, – 4) C. (0, 4) D. (– 4, 0) E. (– 4, 4)

(20) $\sin^2 x + \tan^2 x \sin^2 x =$

A. $\sin^2 x$ B. $\cos^2 x$ C. $\tan^2 x$ D. $\cot^2 x$ E. $\sec^2 x$

(21) If θ is an acute angle and $\sin\theta = x$, then $\sin(270° - 2\theta) =$

A. $2x\sqrt{1-x^2}$ B. $-2x\sqrt{1-x^2}$ C. $2x^2 - 1$ D. $1 - 2x^2$ E. none of these

(22) The exact value of $\sin\frac{7\pi}{9}\cos\frac{4\pi}{9} - \cos\frac{7\pi}{9}\sin\frac{4\pi}{9} =$

A. 1 B. 0 C. $\frac{1}{2}$ D. $\frac{\sqrt{2}}{2}$ E. $\frac{\sqrt{3}}{2}$

(23) If n is a rational number, then $\lim\limits_{h \to 0} \frac{(x+h)^n - x^n}{h} =$

A. n B. x^n C. nx^{n-1} D. $(n-1)x^n$ E. x^{n-1}

(24) The gradient of the curve xy = 1 at (– 1, – 1) is

A. 0 B. – 1 C. 1 D. 2 E. 3

(25) It is given that $y = 2x^3$ and x is changed from 2 to 1.995. Then the approximate change in y is

A. – 0.18 B. – 0.27 C. – 0.12 D. – 0.24 E. – 0.36

(25 marks)

SECTION (B)

(Answer **ALL** questions)

2. If the function $f:R \to R$ is a one-to-one correspondence, then verify that $(f \circ f^{-1})(y) = y$ and $(f^{-1} \circ f)(x) = x$. (3 marks)

(OR)

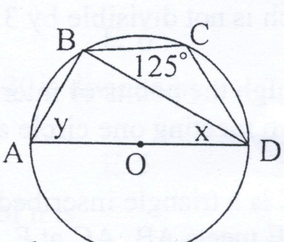
The remainder when $2x^3 + kx^2 + 7$ is divided by $x - 2$ is half the remainder when the same expression is divided by $2x - 1$. Find the value of k. (3 marks)

3. In an A.P. whose first term is -27, the tenth term is equal to the sum of the first nine terms. Calculate the common difference. (3 marks)

(OR)

The first term of a G.P. is a and the common ratio is r. Given that $a = 12r$ and that the sum to infinity is 4, find the third term. (3 marks)

4. In the figure, $\angle BCD = 125°$ find x and y. (3 marks)



5. If $A + B = 45°$, show that $\tan A + \tan B + \tan A \tan B = 1$. (3 marks)

6. Differentiate $f(x) = 1 - 2x^2$ with respect to x at $x = 2$ from the first principles. (3 marks)

SECTION (C)

(Answer any **SIX** questions)

7.(a) The functions f and g are defined for real x by $f(x) = 2x - 1$ and $g(x) = 2x + 3$. Evaluate $(g^{-1} \circ f^{-1})(2)$. (5 marks)

(b) The binary operation $\odot$ on R is defined by $x \odot y = \frac{x^2 + y^2}{2} - xy$, for all real numbers x and y. Show that the operation is commutative, and find the possible values of a such that $a \odot 2 = a + 2$. (5 marks)

8.(a) The expression $px^3 - 5x^2 + qx + 10$ has factor $2x - 1$ but leaves a remainder of -20 when divided by $x + 2$. Find the values of p and q and factorize the expression completely. (5 marks)

(b) In the expansion of $(1 - 2x)^n$, the sum of the coefficients of x and x^2 is 16. Given that n is positive, find the value of n and the coefficient of x^3. (5 marks)

9.(a) Use a graphical method to find the solution set of the inequation $2x(x - 1) < 3 - x$ and illustrate it on the number line. (5 marks)

(b) An A.P. is such that the 5th term is three times the 2nd term. Given further that the sum of the 5th, 6th, 7th and 8th terms is 240, calculate the value of the first term. (5 marks)

[P. T. O.

10.(a) A G.P. of positive terms and an A.P. have the same first term. The sum of their first terms is 1, the sum of their second terms is $\frac{1}{2}$ and the sum of their third terms is 2. Calculate the sum of their fourth terms. (5 marks)

(b) Given that $A = \begin{pmatrix} 5 & 1 \\ a+1 & a \end{pmatrix}$ and det A = 7, find the value of a and then calculate the values of x and y such that $A^2 - xA^{-1} - yI = O$, where I is the unit matrix of order 2. (5 marks)

11.(a) Find the inverse of the matrix $\begin{pmatrix} 7 & -4 \\ -3 & 2 \end{pmatrix}$ and use it to solve the following systems:

$7x - 4y = 13,\ 2y - 3x = -5.$ (5 marks)

(b) How many 3 digit numbers less than 400 can you form by using 1, 2, 3 and 4 without repeating any digit? If one of these numbers is chosen at random, find the probability that it is divisible by 3 but not divisible by 4. Find also the probability that a number which is not divisible by 3. (5 marks)

12.(a) Through the points of intersection of two circles, two straight lines AB and CD are drawn meeting one circle at A, C and the other at B, D. Prove that AC // BD. (5 marks)

(b) ABC is a triangle inscribed in a circle and DE the tangent at A. A line drawn parallel to DE meets AB, AC at F, G respectively. Prove that BFGC is a cyclic quadrilateral. (5 marks)

13.(a) ABC is a right triangle with $\angle A$ the right angle. E and D are points on opposite side of AC, with E on the same side of AC as B, such that ΔACD and ΔBCE are both equilateral. If $\alpha(\Delta BCE) = 2\alpha(\Delta ACD)$, prove that ABC is an isosceles right triangle. (5 marks)

(b) Relative to an origin O the position vectors of the points P and Q are $3\hat{i} + \hat{j}$ and $7\hat{i} - 15\hat{j}$ respectively. Given that R is the point such that $3\overrightarrow{PR} = \overrightarrow{RQ}$, find a unit vector in the direction $\overrightarrow{OR}$. (5 marks)

14.(a) Two acute angles, α and β, are such that $\tan\alpha = \frac{4}{3}$ and $\tan(\alpha + \beta) = -1$. Without evaluating α or β, show that $\tan\beta = 7$, evaluate $\sin\alpha$ and $\sin\beta$. (5 marks)

(b) A ship is 5 km away from a boat in a direction N 37° W and a lighthouse is 12 km away from the boat in a direction S 53° W. Calculate the distance and direction of the ship from the lighthouse. (5 marks)

15.(a) Given that xy = sinx, prove that $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y = 0.$ (5 marks)

(b) Find the approximate change in the volume of a sphere when its radius increases from 2 cm to 2.05 cm. (5 marks)